AGNIHOTRI ENGG. & GATE CLASSES

Unit – II : Topic – Differential Calculus (PD, Eulers Theorem , Errors & App.) **M-I**

Scripting Success Stories

Direction – (Q^n 1 to Q^n 8) : Use PARTIAL DIFFERENTIATION to solve following ?

Q.1) If $u = \tan^{-1}\frac{y}{x} + \sin^{-1}\frac{x}{y}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Q.2) If $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$

Q.3) If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ in term of u is

Q.4) If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$ then prove that : $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

Q.5) If $v = (x^2 + y^2 + z^2)^{-1/2}$, then find the value of: $x\frac{\partial v}{\partial x} + x\frac{\partial v}{\partial y} + y\frac{\partial v}{\partial z}$

Q.6) If $u = x^3 + y^3 - 3xy^2$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ in term of u is

Q.7) If $x^x\, y^y\, z^z = c$, show that at : $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x\log_e x)^{-1}$

Direction – (Q^n 8 to Q^n 11) :Use EULER'S THEOREM to solve following ?

If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that

Q.8) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ **Q.9)** $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} = \sin 4u - \sin 2u$

Q.10) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

Q.11) If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

Direction – (Q^n 12 to Q^n 15) : Using concept of TOTAL DERIVATIVE to solve following ?

Q.12) Find all the partial differentiation coefficient of f(x,y)= xy +sin(x+y)

Q.13) If $x^y + y^x = c$, Find $\frac{dy}{dx}$.

Q.14) If $u = x\log xy$, where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$.

Q.15) If $u = x^2 - y^2 + \sin yz$, where $y = e^x$ and $z = \log_e x$, find $\frac{du}{dx}$

Direction – (Q^n 16 to Q^n 20) :Use concept of APPROXIMATION & ERROR to solve following?

Q.16) Evaluate $\sqrt{99}$ approximately.

Q.17) Evaluate cube root of 127 approximately.

Q.18) Find the percentage error in the area of an ellipse if 1% error is made in measuring the major and minor axes.

Q.19) If H.P. is required to propel a steamer is proportional to cube of its velocity and square of its length ,prove that 2% increase in velocity and 3% increase in length will require an approximately 12% increase in H.P.

Q.20) The period T of a simple pendulum is given by $T=2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible errors upto 1% in I and 2.5% in g.

Questions from Previous year papers (Home Work)

Q.1) If $z(x + y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2 = 4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$. [RGPV – Dec. 2000]

Q.2) If $u = (x^2 + y^2 + z^2)^{-1/2}$; $x^2 + y^2 + z^2 \neq 0$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=-u$,

[RGPV – Feb. 05.Jan./Feb 07]

Q.3) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$. [RGPV – June 01]

Q.4) If $u = \log (x^3 + y^3 + z^3 – 3xyz)$, prove that :

(i) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ (ii) $\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)^2=\frac{9}{(x+y+z)^2}$ (iii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=-\frac{3}{(x+y+z)^2}$

Q.5) If $x^x y^y z^z = c$ then show that $\frac{\partial^2 z}{\partial x\partial y}=-(x\log ex)^{-1}$. [RGPV – Dec. 04]

Q.6) If $u = f(r)$, where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f'(r)+\frac{1}{r}f'(r)$. [RGPV – Dec. 01, June 06]

Q.7) If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$ (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\sin u\cos 2u}{4\cos^3 u}$

Q.8) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$, (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u=\sin 4u-\sin 2u$

Q.9) If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$, prove that: $x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+2xy\frac{\partial^2 u}{\partial x\partial y}=\sin 4u-\sin 2u$

Q.10) If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, show that: $x\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\sin 2u$ [RGPV – June 14]

Q.11) Find the percentage error in the area of rectangle when an error of + 1 percent is made in measuring its length and breadth.

Q.12) If $v = (x^2 + y^2 + z^2)^{-1/2}$, prove that: $x\frac{\partial v}{\partial x}+x\frac{\partial v}{\partial y}+y\frac{\partial v}{\partial z}=-V$

Q.13) If H. P. required to propel a steamer is proportional to cube of its velocity and square of its length; prove that 2% increase in velocity and 8% increase in lengh will require an approximately 12% increase in H.P.

Q.14) If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$ show that: $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$ [RGPV – Dec. 13, June15]