Unit -5 : Topic - Boolean Algebra & Graph Theory **Engg. Maths-I**

Q.1) State whether following are Tautologies or contradiction?

a) $\sim \{(\sim p)\wedge(\sim q)\} \Leftrightarrow (p\vee q)$;

b) $\{p\wedge(p\Rightarrow q)\}\Rightarrow q$;

c) $(p\vee q)\wedge\{p\vee(\sim q)\}\wedge\{(\sim p)\vee q\}\wedge\{(\sim p)\vee(\sim q)\}$

d) $[(p\wedge q)\vee\{q\wedge(\sim r)\}]\Leftrightarrow[\{(\sim p)\wedge r\}\vee\{(\sim q)\wedge(\sim r)\}]$

e) $\sim\{p\wedge(\sim q)\}$;

f) $[(p\Rightarrow q)\wedge(q\Rightarrow r)]\Rightarrow(p\Rightarrow r)$

Q.2) Prove the following

a) $(a + b).(a' + c) = (a.c) + (a'.b)$, $\forall$ a,b,c €B

b) $a.b + a.b' + a'.b + a'.b' = 1$, $\forall$ a,b,c €B

c) $p.q.r + p.q.r'+p.q'.r + p'.q.r = p.q + q.r + r.p$, $\forall$ p,q,r €B

Q.3) State & Prove De Morgan's Theorem ?

Q.4) Express the following functions into disjunctive normal form:

a) $x.y$ **b)** $x + x'.y$ **c)** $f(x,y,z) = x.y' + x.z + x.y$ **d)** $f(x,y,z) = (x + y + z).(xy + x'z)'$

e) $(x.y' + x.z)' + x'$ **f)** $(x + y)(x + y')(x'+z)$ **g)** $(x' + y)'.(x + z)'+(y.z)'$

Q.5) Express the following functions into conjuctive normal form:

a) $f(x,y) = x+x'y$ **b)** $f(x,y,z) = xy' + xz + xy$ **c)** $(x + y + z)(xy + x'z)'$ **d)** $(x + y)(x + y')(x'+ z)$

Answers

Ans.1 a) T b) T c) F d) F e)T f) T

Ans.4 a) $xyz+xyz'$ b) $xy+xy'+x'y$ c) $xyz+xy'z+xyz'+xy'z'$ d) $xy'z+xy'z'+x'yz'$
e) $x'yz+x'yz'+x'y'z+x'y'z'+xyz'$ f) $xyz+xy'z$ g) $xy'z+xy'z'+x'y'z+xyz'+x'yz'+x'y'z'$

Ans.5 a) $(x+y)$ b) $(x+y+z)(x+y'+z)(x+y+z')(x+y'+z')$
c) $(x+y+z)(x'+y'+z)(x'+y'+z')(x+y+z')(x+y'+z')$ d) $(x+y+z)(x+y+z')(x+y'+z)(x+y'+z')(x'+y+z)(x'+y'+z)$

Truth Table of Logical Connectives

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

Some Important Boolean Algebraic Laws

1.	Law of Identity	$A = A$ $\overline{A} = \overline{A}$
2.	Commutative Law	$A \cdot B = B \cdot A$ $A + B = B + A$
3.	Associative Law	$A \cdot (B \cdot C) = A \cdot B \cdot C$ $A + (B + C) = A + B + C$
4.	Idempotent Law	$A \cdot A = A$ $A + A = A$
5.	Double Negative Law	$\overline{\overline{A}} = A$
6.	Complementary Law	$A \cdot \overline{A} = 0$ $A + \overline{A} = 1$
7.	Law of Intersection	$A \cdot 1 = A$ $A \cdot 0 = 0$
8.	Law of Union	$A + 1 = 1$ $A + 0 = A$
9.	DeMorgan's Theorem	$\overline{AB} = \overline{A} + \overline{B}$ $\overline{A + B} = \overline{A}\,\overline{B}$
10.	Distributive Law	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$ $A + (BC) = (A + B) \cdot (A + C)$
11.	Law of Absorption	$A \cdot (A + B) = A$ $A + (AB) = A$
12.	Law of Common Identities	$A \cdot (\overline{A} + B) = AB$ $A + (\overline{A}B) = A + B$

Graph:- A graph is set of objects whose elements or vertices i.e nodes (V) and branches i.e Edges (E)

we denote graph as

$G = (V, E)$

Here

$E = \{e_1, e_2, e_3, e_4\}$

$V = \{v_1, v_2, v_3, v_4\}$

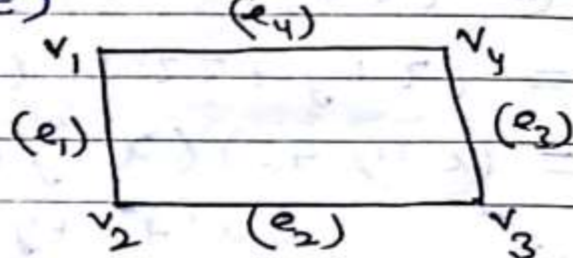


Fig. (a)

Self Loop. (Loop) If an edge starts and end on the same vertex, it is called as self loop.

e_5 is self loop.

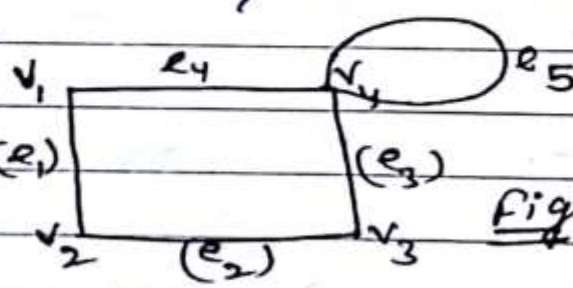
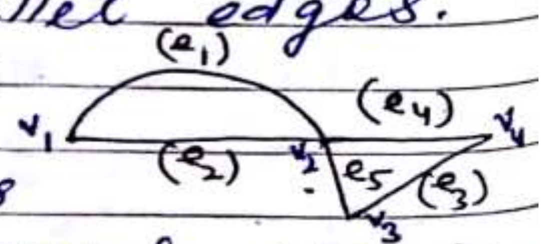


Fig (b)

Parallel Edges:- If two or more than two edges starts and end with the same pair of vertices, then they are said to be parallel edges.

Here e_1 & e_2 share same pair vertices hence they are parallel edges.



Simple Graph:- A graph that has neither self loop nor parallel edge is called a simple graph.

Incident Edges:- In fig. 6 e_3, e_4 & e_5 are incident on vertex V_4.

Adjacent Edge:- Two nonparallel edges which incident on common vertex is called adjacent edge.
In fig. 6 e_2 & e_4 are adjacent edges.

Adjacent vertex:- Two vertex are said to be an adjacent vertex, if there is an edge joining them.
In fig. 6, V_1 & V_2 are adjacent vertex.

Degree of Vertex:- No. of edges incident on a vertex, give the degree of the vertex.

$$\deg(V_4) = 4 \quad \text{in fig 6}$$

where self loop is counted twice.

A vertex with odd degree is called as odd vertex whereas a vertex with even degree is called as Even vertex.

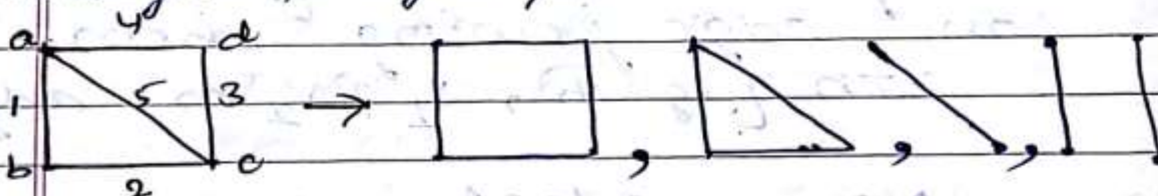
Regular Graph:- A graph with all vertices of same degree, is called Regular graph fig @

Isolated / End vertex:- A vertex of degree zero, is called Isolated or End vertex.

Pendant vertex:- A vertex with degree 1 is called pendant vertex.

Null Graph:- A graph having no edges is called a null graph.

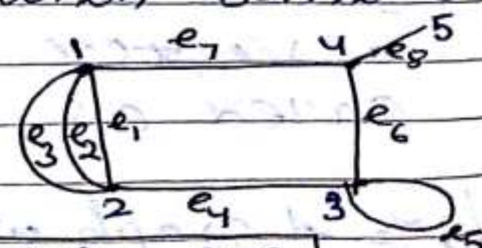
Subgraph:- Subgraph of any graph is obtained by deletion of some vertices or edges from the original graph.



Theorem ⇒ The Sum of the degrees of all vertices in a graph is equal to twice the no. of edges in the graph.

Given:- A graph with some edges & vertices

e.g.



To prove:- $\boxed{\sum \deg(v) = 2e}$

proof:-

$\sum deg(v) = deg(v_1) + deg(v_2) + deg(v_3) + deg(v_4) + deg(v_5)$

$= 4 + 4 + 4 + 3 + 1$

$\sum deg(v) = 16$ —— (i)

No. of edges $= 8$ —— (ii)

from (i) & (ii)

$\sum deg(v) = 2e$ is proved

Theorem:- In any graph, the no. of vertices of odd degree is always even. [RGPV Jan & Jun 2006|02|05]

proof:- Let $G(V, E)$ be graph with

V: set of vertices in G

E: set of Edges in G

Let set of vertices of even degree be V_e and that of odd degree be V_o

If $V_e \cup V_o = V$

$V_e \cap V_o = \phi$

Hence

$$\sum_{v \in V} deg(v) = \sum_{v \in V_e} deg(v) + \sum_{v \in V_o} deg(v) \quad \text{—— (i)}$$

$\because \sum_{v \in V_e} deg(v) = \text{even} = 2K$ —— (ii)

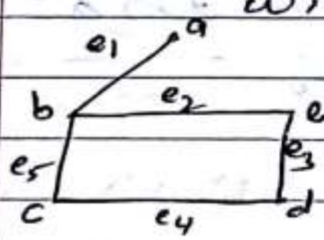
$\because$ we know that $\sum deg(v) = 2e$ —— (iii) from ①, ⑪ & ⑪⑪ we have

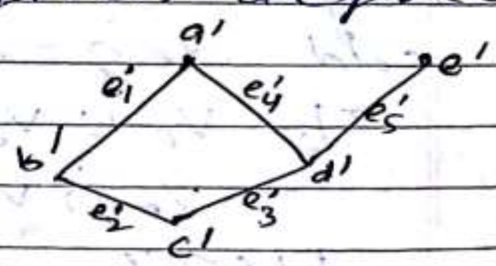
$$2e = 2K + \sum_{v \in V_0} deg(v)$$

$$\sum_{v \in V_0} deg(v) = 2(e-K) = \text{an Even no.}$$

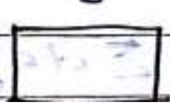
Hence the no. of terms must be even i.e the the no. of vertices of odd degree is always even.

Isomorphic Graph:- If two graphs have same theoritic properties i.e
① They have same no. of edges
⑪ They have same no. of vertices
⑪⑪ They have equal vertices with the given degree

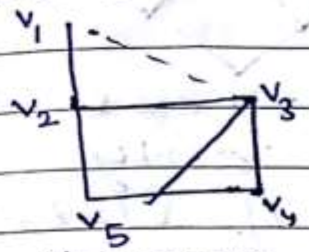




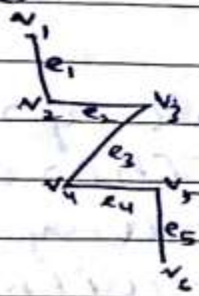
Finite & Infinite Graph:- A graph with finite no. of edges and vertices is called a finite graph otherwise it is an Infinite graph.



Walk:- A walk of graph 'G' is a finite sequence of vertices & edges, beginning & ending with same or diff. vertices, such that each edge is incident with the vertices preceding it. Such that no edge appears more than once. A vertex however may appear more than once.



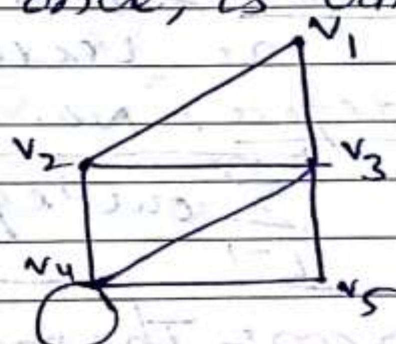
Close walk.



If terminated vertex are diff.

open walk.

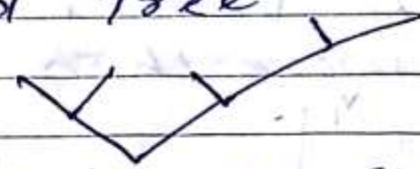
path:- An open walk in which no vertex appear more than once, is called path.



Circuit (cycle):- A closed walk in which no vertex appear more than once is called circuit.
(path) ckt starts and ends with same vertex

Euler Graph:- If a close walk in a graph contains all the edges of the graph then the walk is called Euler line & such graph is called Euler graph.

Trees:- A connected graph without any circuit is called Tree



For finite graph trees are finite

Prop. ① There is only one path b/w every pair of vertices

② Tree with n vertices has $(n-1)$ edges.

In Degree:- No. of edges terminating at vertex.

out degree:- No. of edges beg. at vertex.

$$\boxed{\sum_{i=1}^{n} \text{outdegree}(v_i) = \sum_{i=1}^{n} \text{indegree}(v_i)}$$

Theorem:- The maximum no. of edges with n vertices in a simple graph is $\frac{n(n-1)}{2}$

Proof:- Let $G(V,E)$ has n vertices $v_1, v_2, \ldots v_n$

The vertex v_1 can be joined to $n-1$ vert $v_2, v_3 \ldots v_n$

similarily v_2 can be joined to $(n-2)$ vertices

similarily v_{n-1} — " ——— to only one new edge (v_{n-1}, v_n)

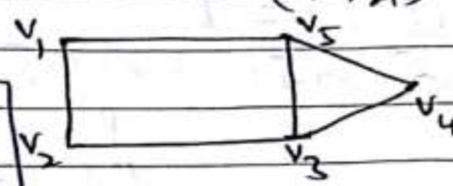
$$\text{Max. Edge} = (n-1)(n-2) + \cdots + 2 + 1 = \frac{1}{2} n(n-1)$$

Matrix Representation of Graph

(a) Adjacency Matrix

$$a_{ij} = \begin{cases} 1 & \text{if there is an edge b/w } i^{th} \text{ \& } j^{th} \text{ vertices} \\ 0 & \text{if there is no edge} \end{cases}$$

Q:- Find Adjacency Matrix (M_A) of graph G



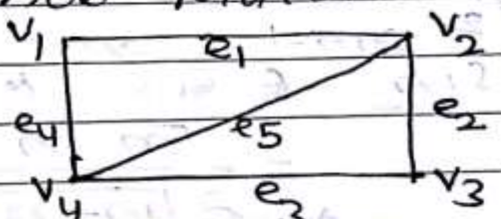
Soln:-

$$M_A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(b) Incidence Matrix :- If $G(V, E)$ then Incidence Matrix is

$$[A]_{n \times m} = [a_{ij}] = \begin{cases} 1 & j^{th} \text{ edge incident on } i^{th} \text{ vertex} \\ 0 & \text{otherwise} \end{cases}$$

Q:- Find the Incidence Matrix 'A' of graph G



Soln

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Note:- For Directed graph, if dirn is toward the vertex consider -1 & similar to earlier case