Unit – I , Topic – Fourier Series & Fourier Transform

Q.1) Find the Fourier series of the function f(x) = e^x

a) In the interval $-\pi < x < \pi$ **b)** In the interval $0 < x < 2\pi$

Q.2) Find the Fourier series of the function f(x) = e^{-x}

a) In the interval $-\pi < x < \pi$ **b)** In the interval $0 < x < 2\pi$

Q.3) Find a series of sines and cosines of multiples of x which will represent $x + x^2$ in interval $-\pi < x < \pi$ and hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

Q.4) Find a Fourier series to represent function $x - x^2$ in $-\pi < x < \pi$ and also deduce $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} \ldots$

Q.5) Find the Fourier expansion for function $f(x) = 2x - x^2$ in (0, 3) and hence deduce $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} \ldots$

Q.6) Expand f(x) = $\sqrt{1 - \cos x}$ in $0 < x < 2\pi$ and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots$

Q.7) Expand f(x) = xsinx for $0 < x < 2\pi$ in a Fourier series?

Q.8) Find the Fourier series for the function $f(x) = \begin{cases} -K, & -\pi < x < \theta \\ K, & 0 < x < \pi \end{cases}$ such that $f(x + 2\pi) = f(x)$

and hence show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$

Q.9) Find the Fourier series of function f(x) = $\begin{cases} 0, & -\pi < x < 0 \\ e^x, & 0 < x < \pi \end{cases}$

Q.10) Find the Fourier series expansion of f(x), when f(x) = $\begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Hence deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots = \frac{\pi^2}{8}$

Q.11) Find the Fourier series to represent the function f(x) = $\begin{cases} 0, & -\pi \le x \le 0 \\ sinx, & 0 \le x \le \pi \end{cases}$

Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots = \frac{1}{2}$

Q.12) Expand the function f(x) in a Fourier series in the interval (-π, π).

f(x) = $\begin{cases} 2x, & 0 \le x \le \pi \\ x, & -\pi \le x \le 0 \end{cases}$

Q.13) Find the Fourier series of f(x) = $\begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$

Q.14) Find the Fourier series to represent the function $f(x)$, given by $f(x) = \begin{cases} x & \text{for } 0 \le x \le \pi \\ 2\pi - x & \text{for } \pi \le x \le 2\pi \end{cases}$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$

Q.15) Obtain the Fourier series for the function given by f(x) = $\begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$

Q.16) Find the Fourier series of the function f(x) = $\begin{cases} 0, & -2 < x < -1 \\ k, & -1 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$

Q.17) Express f(x) = x as a Fourier series in the interval $-\pi < x < \pi$.

Q.18) Find a Fourier series to represent x^2 in the interval (-π, π) & deduce $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$ ...

Q.19) A function is defined as f(x) = $\begin{cases} -x, & -\pi < x \le 0 \\ x, & 0 < x \le \pi \end{cases}$

Show that f(x) = $\frac{\pi}{2} - \frac{4}{\pi}\left[\frac{cosx}{1^2} + \frac{cos3x}{3^2} + \frac{cos5x}{5^2} + \cdots\right]$ and deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

Q.20) Expand as Fourier series to **f**(x) = $\begin{cases} -x+1, & -\pi \le x \le 0 \\ x+1, & 0 \le x \le -\pi \end{cases}$ and deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

Q.21) Expand function f(x) = xsinx as a Fourier series in the interval $-\pi \le x \le \pi$ and deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots = \frac{\pi-2}{4}$

Q.22) Obtain Fourier series in (-π, π) for f(x) = xcosx.

Q.23) For a function f(x) defined by f(x) = $|x|$, $-\pi < x < \pi$. Obtain Fourier series and deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

Q.24) If f(x) = $|cosx|$, expand f(x) as a Fourier series in the interval (-π, π),Deduce $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots = \frac{\pi-2}{4}$

Q.25) If f(x) = $|sinx|$, expand f(x) as a Fourier series in the interval (-π, π),Deduce $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots = \frac{\pi-2}{4}$

Q.26) a) Express f(x) = x as half range sine series in $0 < x < 2$.

b) Express f(x) = x as half range cosine series in $0 < x < 2$.

Q.27) Expand πx - x^2, $0 < x < \pi$ and show that $x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \ldots\ldots\ldots\right)$

Q.28) Obtain half range cosine series for f(x) = $\begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l \end{cases}$ & hnce Deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

Q.29) Express f(x) as the Fourier series of sine term f(x) = $\begin{cases} \frac{1}{4} - x, & 0 < x < 1/2 \\ x - 3/4, & 1/2 < x < 1 \end{cases}$

Q.30) Obtain the half range sine series for f(x) = 2-x for $0 < x < 2$.

Q.31) Find the Fourier Transform of f(x) = $\begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ hence evaluate $\int_0^{\infty} \frac{sinx}{x} dx$.

Q.32) Find the fourier Transform of $\boldsymbol{e}^{-\frac{x^2}{2}}$

Q.33) Find the Fourier transform of f(x) = $\begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ hence evaluate $\int_0^{\infty} \frac{xcosx - sinx}{x^3} cos\frac{x}{2} dx$.

Q.34) Find the Fourier cosine transform of $3e^{-3x}+2e^{-4x}$

Q.35) Find Fourier Cosine transform of sinax

Q.36) Find Fourier Cosine & Sine transform of x^{n-1}

Q.37) Find the Fourier sine transform of $e^{-|x|}$ hence show that $\int_0^{\infty} \frac{xsinmx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0$

Q.38) Find the Fourier cosine transform of f(x)= $\begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$

Q.39) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$

Q.40) Find the Fourier sine transform of $\boldsymbol{e}^{-bx}$

Q.41) Find the Fourier cosine transform of f(x) = $\frac{1}{1+x^2}$

Q.42) If F_C (s) = $\frac{e^{-as}}{s}$, Find f(x)