Direction – (Q^n 1 to Q^n 5)

Solve following using vector differentiation or vector integration

Q.1) If $r = (t^3 + t^2 + t)i + (t^2 + t)j + (t + 1)k$

Find **a)** $\frac{dr}{dt}$ **b)** $\frac{d^2r}{dt^2}$

Q.2) If $r = a\cos t\, i + a\sin t\, j + tk$

Find **a)** $\frac{dr}{dt}$ **b)** $\frac{d^2r}{dt^2}$ **c)** $\left|\frac{d^2r}{dt^2}\right|$

Q.3) If $\vec{r}(t) = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$

Find value of vector $\int_1^2 (\vec{r} \times \frac{d^2\vec{r}}{dt^2})$

Q.4) If $\vec{r}(t) = \begin{cases} 2i - j + 2k & at\ t = 2 \\ 4i - 2j + 3k & at\ t = 3 \end{cases}$

Find $\int_2^3 \vec{r} . \frac{d\vec{r}}{dt} . dt$

Q.5) If $\vec{r} \times d\vec{r} = 0$, prove that $\hat{r}$ is a constant vector?

Direction – (Q^n 6 to Q^n 14)

Solve following using concept of gradient(∇) & Directional Derivative (D.D.)

Q.6) Prove $\nabla r^n = n\, r^{n-2} . \vec{r}$

Q.7) Find Direction Derivative of Scalar point function $x^2yz + 4xz^2$ at (1,-2,-1) in the direction of vector 2i – j – 2k .find max. DD?

Q.8) Find a unit vector normal to the surface $xy^2z^3 = 4$ at (-1,-1,2)?

Q.9) What is the greatest rate of increasing the vector field xyz^3 at point (1,0,3)

Q.10) What is the DD of $xy^2 + yz^3$ at the point (2,-1,1) in the direction of normal to the surface $x \log z - y^2 = -4$ at (-1,2,1).

Q.11) Find the DD of $x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of line PQ , Where Q(5,0,4) .

Q.12) If scalar field $axy^2 = byz = cz^2x^3$ has maximum magnitude in the direction of Z and at (1,2,-1) , Find a,b and c ?

Q.13) Find the DD of scalar field $5x^2y - 5y^2z + 2.5z^2x$ at the point P(1,1,1) in the direction of $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$

Q.14) Find a & b so that surface $ax^2 - byz = (a + 2)x$ will be orthogonal to surface $4x^2y + z^2 = 4$ at (1,-1,2)

Direction – (Q^n 15 to Q^n 25)

Solve following using concept of Div. and curl

Q.15) If Vector field $\vec{F} = xy^2i + 2x^2yz\, j - 3yz^2k$

Find i) div $\vec{F}$ ii) curl $\vec{F}$ at (1,-1,1)

Q.16) If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. Find

i) div. $\vec{F}$ ii) Curl $\vec{F}$

Also show that the Vector Field is irrotational?

Q.17) If $\vec{F} = (x + y + 1)\, i + j - (x + y)\, k$

Show that $\vec{F}$ curl $\vec{F} = 0$

Q.18) Show that $\text{div}(\nabla r^n) = n(n + 1)\, r^{n+2}$

Show that the Vector field is Solenoidal in Q19 & 20

Q.19) $\vec{V} = 3y^4z^2 i + 4x^3z^2 j - 3x^2y^2 k$

Q.20) $\vec{F} = (x + 3y)\, i + (y - 3z)\, j + (x - 2z)\, k$

Q.21) $\vec{F} = (2x + yz)\, i + (4y + 2x)\, j - (6z - xy)\, k$
Prove the vector field is Solenoidal and Irrotational. Also find Scalar Potential of $\vec{F}$.

Q.22) show $\vec{A} = (x^2 + xy^2)\, i + (y^2 + x^2y)\, j$ is irrotational. Also find Scalar Potential?

Q.23) Determine a,b & c. If $\vec{F}$ is Irrotational
$\vec{F} = (x + 2y + az)\, i + (bx - 3y - z)\, j + (4x + cy + 2z)\, k$

Q.24) Prove $\frac{\vec{r}}{r^3}$ is solenoidal.

Q.25) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

Direction – (Q^n 26 to Q^n 34)
Solve Using Vector Integration Theorem

Q.26) Apply Green's theorem to Evaluate
$$\int_C (y - \sin x)dx + \cos x\, dy$$
Where C is Triangular plane enclosed by Lines $y = 0$, $x = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$

Q.27) Apply Green's theorem to Evaluate
$$\int_C (x^2 + y^2)\, i + 2xy\, j$$
Where C is Rectangular plane enclosed by Lines $x = 0$, $x = a$, $y = 0$ and $y = b$

Q.28) Solve using Gauss Divergence theorem
$\vec{F} = x^2 i + zj + yz\, k$, taken over the cube bounded by lines $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ & $z = 1$

Q.29) Evaluate $\oiint \vec{F} ds$, $\vec{F} = 4xi - 2y^2 j + z^2 k$
where S is region of $x^2 + y^2 = 4$, $z = 0$ & $z = 3$

Q.30) Evaluate Using Gauss Divergence thm?
$\oiint \{(x + z)dydz + (y + z)dzdx + (x + z)dxdy\}$
S is region bounded by $x^2 + y^2 + z^2 = 4$

Q.31) Evaluate Using Gauss Divergence thm?
$\oiint x^3 dydz + x^2y dzdx + x^2z dxdy$
S: $z = 0$, $z = b$ & $x^2 + y^2 = a^2$

Q.32) Apply Stokes theorem for Vector field
$\vec{F} = (x^2 + y^2)\, i - 2xy\, j$, integrated around the rectangle in the plane $z = 0$, bounded by lines $x = 0$, $y = 0$, $x = a$ & $y = b$

Q.33) Verify Stokes Theorem for
$\vec{F} = x^2 i + zj + yz\, k$
C: Δ with vertices (0,0,0) , (1,0,0) & (1,1,0)

Q.34) Evaluate using Stokes Theorem
$\int_C \{(x + z)dx + (2x - z)dy + (z + y)dz\}$
C: Δ with vertices (2,0,0) , (0,3,0) & (0,0,6)

Vector Integration Theorems

Gauss Divergence Theorem
$\int_S \bar{F}.d\bar{r} = \int_V curl\, \bar{F} . \bar{n}\, dv$
It gives the relation between double integral and triple Integral
i.e. $\iint \leftrightarrow \iiint$

Green's Theorem
$\int_C P\, dx + Q\, dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy$
It gives the relation between single integral and double Integral
i.e. $\int \leftrightarrow \iint$

Stoke's Theorem
$\int_C \bar{F}.d\bar{r} = \int_S curl\, \bar{F} . \bar{N}\, ds$
It gives the relation between single Integral and double integral
i.e. $\int \leftrightarrow \iint$