# AGNIHOTRI ENGG. & GATE CLASSES

*Scripting Success Stories*

## Questions Sheet

## Unit-3:-Second Order Linear Differential Equation with Variable Coefficient

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**Q.1)** Solve **(3-x)y''-(9-4x)y'+(6-3x)y=0**

**Q.2)** Solve $x^2y''+xy'-y = 0$ , given that **x + 1/x** is one of its integral.

**Q.3)** Solve $xy''+ (1-x)y'- y = e^x$

**Q.4)** Solve $(1-x^2)y''+xy'-y = x(1-x^2)^{3/2}$

**Q.5)** Solve $(x \sin x + \cos x)y'' - x \cos x\, y' + y \cos x = \sin x\,(x \sin x + \cos x)^2$

**Q.6)** Solve $y'' + 2x\, y' +(x^2+1)y = x^3 +3x$

**Q.7)** Solve $y''-x^{-1/2}\, y' +1/4\, x^{-2}\,(x +x^{1/2} -8)y = 0$

**Q.8)** Solve $y''- 2 \tan x\, y' + 5y = \sec x \,.\, e^x$

**Q.9)** Solve $y''+4x.y' +4x^2y = 0$

**Q.10)** Solve $y''- \cot x \,.\, y' - \sin^2 x \,.\, y = \cos x - \cos^3 x$

**Q.11)** Solve $y'' + (1 - 1/x)\, y' + 4x^2\, y\, e^{-2x} = 4(x^2 +x^3)\, e^{-3x}$

**Q.12)** Solve $x.y''-y' - 4x^3y = 8x^3 \sin x^2$

**Q.13)** Solve $y''+ \tan x.y' + y \cos^2 x = 0$

**Q.14)** Solve by method of variation of parameter to the followings

a) $(D^2 +4)y = 4 \tan 2x$

b) $(D^2 +1)y = x \sin x$

c) $x^2.y'' + x.y' - y = x^2.e^x$

d) $(D^2 +1)y = 1/ (1+ \sin x)$

**Q.15)** Solve the following By series method ?

a) $(1+ x^2)y'' +x.y' - y = 0$

b) $( 1- x^2)y'' - x.y' + 4y = 0$

c) $y'' + x.y' + y = 0$

d) $x.y'' + y' - y = 0$

e) $4x.y'' + 2.y' + y =0$

f) $4x.y'' + 2(1-x)y' + y = 0$

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## Theory Sheet

## Unit-3:-Second Order Linear Differential Equation with Variable Coefficient

$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ is called second order differential equation with variable coefficient where P, Q, R are function of x.

**Different Methods of solution of such Differential Equations are**

**Method - 1 :-** when a part of c.f. is known

Rules of Inspection to find u : - If 1 + P + Q = 0, then $y = e^x$ is a part of C.F.
If 1 – P + Q = 0 then $y = e^{-x}$ is a part of C.F.
If P + Qx = 0, then y = x is a part of C.F.

Solve Differential equation to find v : - $\frac{d^2v}{dx^2} + \left(\frac{2}{u}\frac{du}{dx} + P\right)\frac{dv}{dx} = \frac{R}{u}$ Complete Solution is y = uv

**Method - 2 :-** Removal of first derivative

To find u :- $u = e^{-\frac{1}{2}\int p dx}$ To find v Solve Differential equation :- $\frac{d^2v}{dx^2} + Iv = \frac{R}{u}$ where $I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}$

**Method - 3 :-** Change of Independent Variable

$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ can be written as $\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1y = R_1$ Where

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}, \quad Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}, \quad R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$$

**Method - 4 :-** Variation of parameter method

Let $y = c_1y_1 + c_2y_2$ be the known complimentary function (C. F.)

Then P. I. = $-y_1\int\frac{y_2R}{W}dx + y_2\int\frac{y_1R}{W}dx$ Where $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ Complete Solution is y = C.F. + P.I.

**Method - 5 :-** Series Solution method Let differential equation is $P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$

Here $P_0(x)$, $P_1(x)$, $P_2(x)$ are polynomial of x.
If $P_0(x)$ is 0 at x = 0 i.e. P(0) = 0 then x = 0 is **singular point**
If $P_0(x) \neq 0$ at x = 0 i.e. $P(0) \neq 0$ then x = 0 is said to be **ordinary point**.

Ordinary Point Solution - $y = a_0 + a_1x + a_2x^2 + a_3x^3 + ....a_rx^r + ....\infty$ then Find $\frac{dy}{dx}$ ; and $\frac{d^2y}{dx^2}$ & put in Eqn.